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Master equation theory applied to the redistribution of polarized radiation in the weak radiation field limit

V. The two-term atom
(Corrigendum)

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A sign factor was lacking in the expressions of the redistribution matrix in the case of incomplete Paschen-Back effect, Eqs. (A.1) and (A.6) of Bommier (2017). This sign factor is unity in the absence of incomplete Paschen-Back effect. A $(2I + 1)$ denominator was also missing in Eqs. (A6) and (40), and some typos occurred in Eq. (A.6). The correct formulæ are provided below.

The corrected Eq. (A.1) is:

$$
\begin{aligned}
\mathcal{R}_{ij}\left(\nu,\nu_1,\boldsymbol{\Omega},\boldsymbol{\Omega}_1;\boldsymbol{B}\right) = & \sum_{J_u \bar{J}_u J_u^* M_u J_u' \bar{J}_u' J_u'^* M_u' J_\ell \bar{J}_\ell J_\ell^* M_\ell J_\ell' \bar{J}_\ell' J_\ell'^* M_\ell' K K' Q} \int f(\boldsymbol{v}) \mathrm{d}^3\boldsymbol{v}\, (-1)^Q \mathcal{T}^{K'}_{-Q}(j,\boldsymbol{\Omega}_1) \mathcal{T}^{K}_{Q}(i,\boldsymbol{\Omega}) \\
& \times 3 \frac{2L_u + 1}{2S + 1} \sqrt{(2K+1)(2K'+1)}\, (-1)^{M_\ell - M_\ell'} (-1)^{J_u + \bar{J}_u + J_u' + \bar{J}_u'} (-1)^{J_\ell + \bar{J}_\ell + J_\ell' + \bar{J}_\ell'} \\
& \times \sqrt{(2J_u+1)(2\bar{J}_u+1)(2J_u'+1)(2\bar{J}_u'+1)(2J_\ell+1)(2\bar{J}_\ell+1)(2J_\ell'+1)(2\bar{J}_\ell'+1)} \\
& \times C^{J_u}_{J_u^* M_u}(B)\, C^{\bar{J}_u}_{J_u^* M_u}(B)\, C^{J_u'}_{J_u'^* M_u'}(B)\, C^{\bar{J}_u'}_{J_u'^* M_u'}(B)\, C^{J_\ell}_{J_\ell^* M_\ell}(B)\, C^{\bar{J}_\ell}_{J_\ell^* M_\ell}(B)\, C^{J_\ell'}_{J_\ell'^* M_\ell'}(B)\, C^{\bar{J}_\ell'}_{J_\ell'^* M_\ell'}(B) \\
& \times \begin{Bmatrix} J_u & 1 & J_\ell \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} J_u' & 1 & \bar{J}_\ell \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} \bar{J}_u & 1 & J_\ell' \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} \bar{J}_u' & 1 & \bar{J}_\ell' \\ L_\ell & S & L_u \end{Bmatrix} \\
& \times \begin{pmatrix} J_u & 1 & J_\ell \\ -M_u & p & M_\ell \end{pmatrix} \begin{pmatrix} J_u' & 1 & \bar{J}_\ell \\ -M_u' & p' & M_\ell \end{pmatrix} \begin{pmatrix} \bar{J}_u & 1 & J_\ell' \\ -M_u & p''' & M_\ell' \end{pmatrix} \begin{pmatrix} \bar{J}_u' & 1 & \bar{J}_\ell' \\ -M_u' & p'' & M_\ell' \end{pmatrix} \\
& \times \begin{pmatrix} 1 & 1 & K' \\ -p & p' & Q \end{pmatrix} \begin{pmatrix} 1 & 1 & K \\ -p''' & p'' & Q \end{pmatrix} \\
& \times \left\{ \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E + \dfrac{\mathrm{i}\Delta E_{M_u M_u'}}{\hbar}} \delta\left(\tilde{\nu} - \tilde{\nu}_1 - \nu_{M_\ell M_\ell'}\right) \left[\frac{1}{2}\Phi_{ba}\left(\nu_{M_u' M_\ell} - \tilde{\nu}_1\right) + \frac{1}{2}\Phi^*_{ba}\left(\nu_{M_u M_\ell} - \tilde{\nu}_1\right) \right] \right. \\
& + \left[\frac{\Gamma_R}{\Gamma_R + \Gamma_I + \dfrac{\mathrm{i}\Delta E_{M_u M_u'}}{\hbar}} - \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E + \dfrac{\mathrm{i}\Delta E_{M_u M_u'}}{\hbar}} \right] \\
& \left. \times \left[\frac{1}{2}\Phi_{ba}\left(\nu_{M_u' M_\ell} - \tilde{\nu}_1\right) + \frac{1}{2}\Phi^*_{ba}\left(\nu_{M_u M_\ell} - \tilde{\nu}_1\right) \right] \left[\frac{1}{2}\Phi_{ba}\left(\nu_{M_u' M_\ell'} - \tilde{\nu}\right) + \frac{1}{2}\Phi^*_{ba}\left(\nu_{M_u M_\ell'} - \tilde{\nu}\right) \right] \right\} .
\end{aligned}
\quad \text{(A.1)}
$$

The corrected Eq. (A.6) is:

$$
\begin{aligned}
\mathcal{R}_{ij}\left(\nu,\nu_1,\mathbf{\Omega},\mathbf{\Omega}_1;\boldsymbol{B}\right) \;=\; & \sum_{J_u\bar{J}_uJ_u^*F_u\bar{F}_uF_u^*M_uJ'_u\bar{J}'_uJ'^*_uF'_u\bar{F}'_uF'^*_uM'_uJ_\ell\bar{J}_\ell J_\ell^*F_\ell\bar{F}_\ell F_\ell^*M_\ell J'_\ell\bar{J}'_\ell J'^*_\ell F'_\ell\bar{F}'_\ell F'^*_\ell M'_\ell KK'Q} \\
& \times \int f(\boldsymbol{v})\mathrm{d}^3\boldsymbol{v}\,(-1)^Q\mathcal{T}^{K'}_{-Q}(j,\mathbf{\Omega}_1)\mathcal{T}^K_Q(i,\mathbf{\Omega}) \\
& \times 3\,\frac{2L_u+1}{(2I+1)(2S+1)}\sqrt{(2K+1)(2K'+1)}\,(-1)^{M_\ell-M'_\ell} \\
& \times (-1)^{J_u+\bar{J}_u+J'_u+\bar{J}'_u}(-1)^{J_\ell+\bar{J}_\ell+J'_\ell+\bar{J}'_\ell}(-1)^{F_u+\bar{F}_u+F'_u+\bar{F}'_u}(-1)^{F_\ell+\bar{F}_\ell+F'_\ell+\bar{F}'_\ell} \\
& \times \sqrt{(2J_u+1)(2\bar{J}_u+1)(2J'_u+1)(2\bar{J}'_u+1)(2J_\ell+1)(2\bar{J}_\ell+1)(2J'_\ell+1)(2\bar{J}'_\ell+1)} \\
& \times \sqrt{(2F_u+1)(2\bar{F}_u+1)(2F'_u+1)(2\bar{F}'_u+1)(2F_\ell+1)(2\bar{F}_\ell+1)(2F'_\ell+1)(2\bar{F}'_\ell+1)} \\
& \times C^{J_u}_{J_u^*M_{J_u}(F_u^*M_u)}(B)\,C^{\bar{J}_u}_{J_u^*M_{J_u}(F_u^*M_u)}(B)\,C^{J'_u}_{J'^*_uM'_{J_u}(F'^*_uM'_u)}(B)\,C^{\bar{J}'_u}_{J'^*_uM'_{J_u}(F'^*_uM'_u)}(B) \\
& \times C^{J_\ell}_{J_\ell^*M_{J_\ell}(F_\ell^*M_\ell)}(B)\,C^{\bar{J}_\ell}_{J_\ell^*M_{J_\ell}(F_\ell^*M_\ell)}(B)\,C^{J'_\ell}_{J'^*_\ell M'_{J_\ell}(F'^*_\ell M'_\ell)}(B)\,C^{\bar{J}'_\ell}_{J'^*_\ell M'_{J_\ell}(F'^*_\ell M'_\ell)}(B) \\
& \times C^{F_u}_{F_u^{**}(J_uM_{J_u})M_u}(B)\,C^{\bar{F}_u}_{\bar{F}_u^{**}(\bar{J}_uM_{J_u})M_u}(B)\,C^{F'_u}_{F'^{**}_u\left(J'_uM'_{J_u}\right)M'_u}(B)\,C^{\bar{F}'_u}_{\bar{F}'^{**}_u\left(\bar{J}'_uM'_{J_u}\right)M'_u}(B) \\
& \times C^{F_\ell}_{F_\ell^{**}(J_\ell M_{J_\ell})M_\ell}(B)\,C^{\bar{F}_\ell}_{\bar{F}_\ell^{**}(\bar{J}_\ell M_{J_\ell})M_\ell}(B)\,C^{F'_\ell}_{F'^{**}_\ell\left(J'_\ell M'_{J_\ell}\right)M'_\ell}(B)\,C^{\bar{F}'_\ell}_{\bar{F}'^{**}_\ell\left(\bar{J}'_\ell M'_{J_\ell}\right)M'_\ell}(B) \\
& \times \begin{Bmatrix} J_u & 1 & J_\ell \\ L_\ell & S & L_u \end{Bmatrix}\begin{Bmatrix} J'_u & 1 & \bar{J}_\ell \\ L_\ell & S & L_u \end{Bmatrix}\begin{Bmatrix} \bar{J}_u & 1 & J'_\ell \\ L_\ell & S & L_u \end{Bmatrix}\begin{Bmatrix} \bar{J}'_u & 1 & \bar{J}'_\ell \\ L_\ell & S & L_u \end{Bmatrix} \\
& \times \begin{Bmatrix} F_u & 1 & F_\ell \\ J_\ell & I & J_u \end{Bmatrix}\begin{Bmatrix} F'_u & 1 & \bar{F}_\ell \\ \bar{J}_\ell & I & J_u \end{Bmatrix}\begin{Bmatrix} \bar{F}_u & 1 & F'_\ell \\ J_\ell & I & \bar{J}_u \end{Bmatrix}\begin{Bmatrix} \bar{F}'_u & 1 & \bar{F}'_\ell \\ \bar{J}_\ell & I & \bar{J}_u \end{Bmatrix} \\
& \times \begin{pmatrix} F_u & 1 & F_\ell \\ -M_u & p & M_\ell \end{pmatrix}\begin{pmatrix} F'_u & 1 & \bar{F}_\ell \\ -M'_u & p' & M_\ell \end{pmatrix}\begin{pmatrix} \bar{F}_u & 1 & F'_\ell \\ -M_u & p''' & M'_\ell \end{pmatrix}\begin{pmatrix} \bar{F}'_u & 1 & \bar{F}'_\ell \\ -M'_u & p'' & M'_\ell \end{pmatrix} \\
& \times \begin{pmatrix} 1 & 1 & K' \\ -p & p' & Q \end{pmatrix}\begin{pmatrix} 1 & 1 & K \\ -p''' & p'' & Q \end{pmatrix} \\
& \times \Bigg\{ \frac{\Gamma_R}{\Gamma_R+\Gamma_I+\Gamma_E+\dfrac{\mathrm{i}\Delta E_{M_uM'_u}}{\hbar}}\,\delta\left(\tilde{\nu}-\tilde{\nu}_1-\nu_{M_\ell M'_\ell}\right)\left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_uM_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_uM_\ell}-\tilde{\nu}_1\right)\right] \\
+ & \left[\frac{\Gamma_R}{\Gamma_R+\Gamma_I+\dfrac{\mathrm{i}\Delta E_{M_uM'_u}}{\hbar}}-\frac{\Gamma_R}{\Gamma_R+\Gamma_I+\Gamma_E+\dfrac{\mathrm{i}\Delta E_{M_uM'_u}}{\hbar}}\right] \\
\times & \left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_uM_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_uM_\ell}-\tilde{\nu}_1\right)\right]\left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_uM'_\ell}-\tilde{\nu}\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_uM'_\ell}-\tilde{\nu}\right)\right]\Bigg\} .
\end{aligned}
\tag{A.6}
$$

This equation is in excellent agreement as for the Racah algebra with Eq. (30) of Casini et al. (2014). The product of two coefficients $C^J_{J^*M_J(F^*M)}(B)\,C^F_{F^{**}(JM_J)M}(B)$ is equal to the coefficient $C^{JF}_\mu(M)$ of Casini et al. (2014), because these coefficients all result from matrix diagonalization, performed in one step (FS + HFS) in Casini et al. (2014) and in two steps (FS and HFS) in our case. A similar coefficient is visible in Eq. (3.58) of Landi Degl'Innocenti & Landolfi (2004).

The following equation replaces Eq. (40) of Bommier (2017), by introducing the $(2I+1)$ denominator

$$
\begin{aligned}
\mathcal{R}_{ij}(\nu,\nu_1,\mathbf{\Omega},\mathbf{\Omega}_1;\boldsymbol{B}=\mathbf{0}) \quad = & \sum_{J_uF_uJ'_uF'_uJ_\ell F_\ell J'_\ell F'_\ell KQ} \int f(\boldsymbol{v})\mathrm{d}^3\boldsymbol{v}\,(-1)^Q \mathcal{T}^K_{-Q}(j,\mathbf{\Omega}_1)\mathcal{T}^K_Q(i,\mathbf{\Omega}) \\
& \times 3\frac{2L_u+1}{(2I+1)(2S+1)}(2J_u+1)(2J'_u+1)(2J_\ell+1)(2J'_\ell+1)(2F_u+1)(2F'_u+1)(2F_\ell+1)(2F'_\ell+1)\,(-1)^{F_\ell-F'_\ell} \\
& \times \begin{Bmatrix} J_u & 1 & J_\ell \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} J'_u & 1 & J_\ell \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} J_u & 1 & J'_\ell \\ L_\ell & S & L_u \end{Bmatrix} \begin{Bmatrix} J'_u & 1 & J'_\ell \\ L_\ell & S & L_u \end{Bmatrix} \\
& \times \begin{Bmatrix} F_u & 1 & F_\ell \\ J_\ell & I & J_u \end{Bmatrix} \begin{Bmatrix} F'_u & 1 & F_\ell \\ J_\ell & I & J'_u \end{Bmatrix} \begin{Bmatrix} F_u & 1 & F'_\ell \\ J_\ell & I & J_u \end{Bmatrix} \begin{Bmatrix} F'_u & 1 & F'_\ell \\ J_\ell & I & J'_u \end{Bmatrix} \\
& \times \begin{Bmatrix} K & F_u & F'_u \\ F_\ell & 1 & 1 \end{Bmatrix} \begin{Bmatrix} K & F_u & F'_u \\ F'_\ell & 1 & 1 \end{Bmatrix} \\
& \times \Bigg\{ \frac{\Gamma_R}{\Gamma_R+\Gamma_I+\Gamma_E+\frac{\mathrm{i}\Delta E_{F_uF'_u}}{\hbar}}\,\delta\left(\tilde{\nu}-\tilde{\nu}_1-\nu_{F_\ell F'_\ell}\right)\left[\frac{1}{2}\Phi_{ba}\left(\nu_{F'_uF_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{F_uF_\ell}-\tilde{\nu}_1\right)\right] \\
& + \left[\frac{\Gamma_R}{\Gamma_R+\Gamma_I+\frac{1}{2}\left[D^{(K)}(\alpha_uF_u)+D^{(K)}(\alpha_uF'_u)\right]+\frac{\mathrm{i}\Delta E_{F_uF'_u}}{\hbar}} - \frac{\Gamma_R}{\Gamma_R+\Gamma_I+\Gamma_E+\frac{\mathrm{i}\Delta E_{F_uF'_u}}{\hbar}}\right] \\
& \times \left[\frac{1}{2}\Phi_{ba}\left(\nu_{F'_uF_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{F_uF_\ell}-\tilde{\nu}_1\right)\right]\left[\frac{1}{2}\Phi_{ba}\left(\nu_{F'_uF'_\ell}-\tilde{\nu}\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{F_uF'_\ell}-\tilde{\nu}\right)\right]\Bigg\}
\end{aligned}
\quad , \quad (40)
$$

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